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Existence of solutions for quasilinear random impulsive neutral differential evolution equation

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Abstract. This paper deals with the existence of solutions for quasilinear random impulsive neutral functional differential evolution equation in Banach spaces and the results are derived by using the analytic semigroup theory, fractional powers of operators and the Schauder fixed point approach. An application is provided to illustrate the theory.

Keywords: Quasilinear differential equation; Analytic semigroup; Random impulsive neutral differential equation; Fixed point theorem

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1. Introduction

In many fields of science and engineering the accurate analysis, design and assessment of systems subjected to realistic environments must take into account the potential of white noise random forces in the system properties. Randomness is acquired by a dynamical system from outside in the form of certain random action. It is this action that causes the randomness of change in the state of the system and in many other quantities determined which enables us to represent a dynamical system as a certain transformation of random inputs into random

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outputs. Randomness is intrinsic to the mathematical formulation of many phenomena, such as: fluctuations in the stock market, noise in population systems, communication networks in observed signals, etc.

The use of deterministic equations that ignore the randomness of the parameters or replace them by their mean values can result in gross errors. It is more important to consider the case when the perturbation term is rather widely impulsive in character and it is natural to expect such a situation in biological systems such as heart beats, blood flows, pulse frequency modulated systems, models for biological neural nets and automatic control problems. Therefore, perturbations of impulsive type are more realistic. This makes the study interesting.

In this paper we consider the following quasilinear random impulsive neutral differential evolution equation in a Banach space $\mathbb{X}$

$$\left.\begin{aligned} [u(t)+g(t,u(t))]' + A(t,u)u(t) &= f(t,u(t)), \quad t \in [0,T], \ t \neq \xi_k, \\ u(0) &= u_0, \\ u(\xi_k) &= b_k(\tau_k)u(\xi_k^-), \ k = 1,2,\ldots, \end{aligned}\right\} \tag{1.1}$$

where $-A(t,u)$ is the infinitesimal generator of an analytic semigroup of operators in a Banach space $\mathbb{X}$.

Now we make the system (1.1) more precise: The function $f : J \times \mathbb{X} \to \mathbb{X}$ is uniformly bounded and continuous in all of its arguments and $u_0 \in \mathbb{X}$ and $g : J \times \mathbb{X} \to \mathbb{X}$. Take $J = [0,T]$, $T \in \mathbb{R}$ is any constant. Assume that Δ be a non-empty set and τ_k is a random variable defined from Δ to $D_k \equiv (0,d_k)$, for $k = 1,2,\ldots$ where $0 < d_k < +\infty$. Also assume that τ_i and τ_j are independent from each other as $i \neq j$ for $i,j = 1,2,\ldots$. Let $b_k : D_k \to \mathbb{R}$, for each $k = 1,2,\ldots$; $\xi_0 = t_0$ and $\xi_k = \xi_{k-1} + \tau_k$ for $k = 1,2,\ldots$; here $t_0 \in J$ is an arbitrary real number. Obviously, $t_0 = \xi_0 < \xi_1 < \cdots < \lim_{k\to\infty}\xi_k = \infty$; $u(\xi_k^-) = \lim_{t\to\xi_k} u(t)$ according to their paths with the norm $\|u\| = \sup_{0\le s\le T}|u(s)|$, for each t satisfying $0 \le t \le T$, $\|\cdot\|$ is any given norm in $\mathbb{X}$.

The problem of existence of solutions for quasilinear equation in Banach spaces has been studied by many authors. Furuya [7] and Kato [8] studied the non-homogeneous quasilinear evolution equation and the analyticity of solution in 1980's. Bahuguna [3,4] proved the existence, uniqueness and continuous dependence of a strong and local solutions to the quasilinear integrodifferential equations and also the regularity of solutions to the quasilinear equations. Oka and Tanaka [10] implemented the existence of classical solutions of abstract quasilinear integrodifferential equations. Kato [9] concentrated on the applications to PDE for the quasilinear evolution equations.

Many researchers have investigated the qualitative properties of fixed-type impulses in [15,16]. Radhakrishnan et al. [13] studied the impulsive neutral functional evolution integrodifferential systems with infinite delay. Wu et al. [17] first introduced the existence and uniqueness of solutions to random impulsive functional differential equations. Anguraj et al. [2] proved the existence and exponential stability of semilinear functional differential equations with random impulses under non-uniqueness. Yong and Wu [20] investigated the solutions of stochastic differential equations with Random impulse using Lipschitz condition. Wu et al. [19,18] discussed the boundedness and exponential stability of differential equations with random impulses.

Recently, Balachandran and Park [5] investigated the existence of solutions of quasilinear integrodifferential evolution equations by Schauder fixed point approach. Balachandran and Uchiyama [6] discussed the existence of solutions to quasilinear integrodifferential equation with nonlocal condition. Radhakrishnan [12] investigated the existence of quasilinear neutral impulsive integrodifferential equations in Banach space. Radhakrishnan et al. [14] discussed about the various types of equations such as semilinear, quasilinear and its controllability results. For the application of analytic semigroups to related quasilinear evolution equations we refer to Amann [1] and references therein. The study of random impulsive differential equations has attracted a great attention nowadays.

Motivated by this fact, in this paper we make a first attempt to study the existence and uniqueness results for random impulsive quasilinear neutral functional differential evolution equation by using the fixed point approach.

2. Preliminaries

Consider the Cauchy problem for the quasilinear initial value problem

$$\left.\begin{aligned} u'(t) + A(t,u)u(t) &= f(t,u(t)), \quad 0 \le s \le t \le T, \\ u(s) &= v, \end{aligned}\right\} \tag{2.1}$$

with an operator $-A(t,u)$ which is the infinitesimal generator of an analytic semigroup on a Banach space $\mathbb{X}$. We make the following assumptions.

(E1) The domain $D(A(t,u(t))) = D$ of $A(t,u(t))$, $0 \le t \le T$ is dense in $\mathbb{X}$.

(E2) For $t \in J$, the resolvent $R(\lambda; A(t,u(t))) = (\lambda I - A(t,u(t)))^{-1}$, of $A(t,u(t))$ exists for all λ with $Re\lambda \le 0$ and there is a constant C such that for $Re\lambda \le 0$, $t \in J$,

$$\|R(\lambda; A(t,u(t)))\| \le C[\|\lambda\| + 1]^{-1}.$$

(E3) There exists constants L and $0 \le \alpha \le 1$ such that for $t, s \in J$,

$$\|A(t,u(t)) - A(s,u(t))\| \le L|t-s|^{\alpha}.$$

Theorem 2.1 ([11]). *Let $\mathbb{B} \subset \mathbb{X}$ and $A(t,b)$, $(t,b) \in I \times \mathbb{B}$ be a family of operators satisfying (E1)–(E3), there is a unique evolution system $\mathcal{S}_u(t,s)$ on $0 \le s \le t \le T$, satisfying*

(i) *$S_u(t,s) \le M_0$, for $0 \le s \le t \le T$.*

(ii) *For $0 \le s \le t \le T$, $S_u(t,s) : \mathbb{X} \to D$ and $t \to S_u(t,s)$ is strongly differentiable in $\mathbb{X}$. The derivative $\frac{\partial}{\partial t} S_u(t,s) \in \mathbb{B}(\mathbb{X})$ and it is strongly continuous on $0 \le s \le t \le T$. Moreover,*

$$\frac{\partial}{\partial t} S_u(t,s) + A(t,u)S_u(t,s) = 0, \quad \textit{for } 0 \le s \le t \le T,$$

$$\|\frac{\partial}{\partial t} S_u(t,s)\| = \|A(t,u(s))S_u(t,s)\| \le M_0(t-s)^{-1} \quad \textit{and}$$

$$\|A(t,u)S_u(t,s)A^{-1}(s,u)\| < M_0, \quad \textit{for } 0 < s < t < T.$$

(iii) *For every $w \in D$, $t \in J$, $S_u(t,s)w$ is differentiable with respect to s on $0 \le s \le t \le T$,*

$$\frac{\partial}{\partial s} S_u(t,s)w = -S_u(t,s)A(s,u(s))w.$$

(iv) *$S_u(t,s)$ is strongly continuous for $0 \le s \le t \le T$ and*

$$S_u(t,r) = S_u(t,s)S_u(s,r), \quad \text{for } r \le s \le t,$$
$$S_u(t,t) = I.$$

From the condition (E2) and the fact that D is dense in $\mathbb{X}$ implies that for every $t \in [0,T]$, $-A(t,u(t))$ is the infinitesimal generator of an analytic semigroup.

Define the classical solution of (2.1) as a function $u : [s,T] \to \mathbb{X}$ which is continuous for $s \le t \le T$, continuously differentiable for $s < t \le T$, $u(t) \in D$ for $s < t \le T$, $u(s) = v$ and $u'(t) + A(t,u)u(t) = f(t,u(t))$ holds for $s < t \le T$. We will call the function $u(t)$ as a solution of the initial value problem (2.1) if it is a classical solution of the problem.

Theorem 2.2. *Let $A(t,u(t)), 0 \le t \le T$ satisfy the conditions (E1)–(E3) and let $S_u(t,s)$ be the evolution system in Theorem 2.1 If f is Holder continuous on $[0,T]$, then the initial value problem (2.1) has, for every $v \in \mathbb{X}$, a unique solution $u(t)$ given by*

$$u(t) = S_u(t,s)v + \int_s^t S_u(t,\tau)f(\tau,u(\tau))d\tau.$$

The proofs of the above theorems can be found in (Ref. [11]).

A function $u \in C(I : \mathbb{X})$ such that $u(t) \in D(A(t,u(t)))$ for $t \in (0,a]$, $u \in C^1((0,a] : \mathbb{X})$ and satisfied (2.1) in $\mathbb{X}$ is called a classical solution of (2.1) on I. Further there exists a constant $K > 0$ such that for every $u, v \in C(I : \mathbb{X})$ with values in B and every $w \in \mathbb{E}$ we have

$$\|S_u(t,s)w - S_v(t,s)w\| \le K\|w\|_Y \int_s^t \|u(\tau) - v(\tau)\|d\tau.$$

3. Existence and uniqueness

In this section, we discuss about the existence of solutions for quasilinear differential equation with random impulsive condition by using fractional powers of operators and the Schauder fixed point approach.

Let $r > 0$ and take $\mathbb{B}_r = \{v \in \mathbb{X}; \|v\|_{\mathcal{PC}} < r\}$, and assume the following conditions.

(A1) The operator $A_0 = A(0,u_0)$ is a closed operator with domain D, dense in $\mathbb{X}$ and $\|(\lambda I - A_0)^{-1}\| \le C_1[\|\lambda\| + 1]^{-1}$, for all λ with $Re\lambda \le 0$.

(A2) The operator A_0^{-1} is a completely continuous operator in $\mathbb{X}$.

(A3) For some $\alpha \in [0,1)$ and for any $v \in \mathbb{B}_r$, the operator $A(t, A_0^{-\alpha}v)$ is well defined on D for all $t \in J$. Also for any $t, \tau \in J$ and for $v, w \in \mathbb{B}_r$,

$$\|[A(t, A_0^{-\alpha}v) - A(\tau, A_0^{-\alpha}w)]A_0^{-1}\| \le C_2[|t-\tau|^{\epsilon} + \|v-w\|^{\rho}],$$

where $0 < \epsilon \le 1, 0 < \rho \le 1$.

(A4) For every $t, \tau \in J$ and $v, w \in \mathbb{B}_r$,

$$\|g(t, A_0^{-\alpha}v) - g(\tau, A_0^{-\alpha}w)\| \le C_3[|t-\tau|^{\epsilon} + \|v-w\|^{\rho}].$$

(A5) For every $t, \tau \in J$ and $v, w \in \mathbb{B}_r$,

$$\|f(t, A_0^{-\alpha}v) - f(\tau, A_0^{-\alpha}w)\| \le C_4[|t-\tau|^{\epsilon} + \|v-w\|^{\rho}].$$

(A6) $x_0 \in D(A_0^\beta)$, for any $\beta > \alpha$ and $\|A_0^\alpha x_0\| < r$.
(A7) $\max_{i,k}\{\prod_{j=i}^{k} \|b_j(\tau_j)\|\} \leq M_1$.

From these assumptions, we have the following [Kato]

(K1) $\|A(t,u)^\alpha S_u(t,s)\| \leq (\beta-\alpha)^{-1} N_1 (t-s)^{-\alpha}$, for $N_1 > 0, 0 \leq \alpha < \beta$.
(K2) $\|A(0,u)^\alpha A(t,u)^{-\alpha}\| \leq M_2$, for $M_2 > 0, 0 \leq t \leq T$.
(K3) $\|A_0^\alpha [S_u(t,0) - S_u(\tau,0)] A(0)^{-\beta}\| \leq C_5 |t-\tau|^{\beta-\alpha}$, for $t, \tau \in J, 0 \leq \alpha < \beta$.
(K4) $\|A_0^\alpha [S_u(t,\xi_k) - S_u(\tau,\xi_k)]\| \leq C_6 |t-\tau|^{1-\alpha} (\tau - \xi_k)^{-1}$, for every $k = 1, 2, \ldots$.

For convenience, choose $f_u(t) = f(t, A_0^{-\alpha} u(t))$ and $g_u(t) = g(t, A_0^{-\alpha} u(t))$.

Then it follows that the functions $f_u(t)$, $g_u(t)$ are Holder continuous such that

$$\|f_u(t) - f_u(\tau)\| \leq C_7 |t-\tau|^\mu, \text{ for } t, \tau \in J$$

$$\|g_u(t) - g_u(\tau)\| \leq C_8 |t-\tau|^\mu, \text{ for } t, \tau \in J,$$

where $\mu = \min\{\epsilon, \eta\rho\}$, $0 < \eta < \beta - \alpha$.

Lemma 3.1. *Let the functions $f_u(t)$ and $g_u(t)$ are continuous on $[0,T]$. Then for any $0 \leq t_2 \leq t_1 \leq T$, $0 \leq \alpha < \beta$, the following inequality holds.*

$$\begin{aligned}
&\|A_0^\alpha \Big[\int_0^{t_1} S_u(t_1,s) f_u(s) ds - \int_0^{t_2} S_u(t_2,s) f_u(s) ds\Big]\| \\
&\quad \leq C_9 |t_1 - t_2|^{1-\alpha} \Big(|log(t_1 - t_2)| + 1\Big) \\
&\|A_0^\alpha \Big[\int_0^{t_1} S_u(t_1,s) A(s,u) g_u(s) ds - \int_0^{t_2} S_u(t_2,s) A(s,u) g_u(s) ds\Big]\| \\
&\quad \leq C_{10} |t_1 - t_2|^{1-\alpha} \Big(|log(t_1 - t_2)| + 1\Big).
\end{aligned}$$

Let us also define the operator $A_u(t) = A(t, A_0^{-\alpha} u(t))$ such that for $u(0) = A_0^\alpha u_0$,

$$A_u(0) = A(0, A_0^{-\alpha} u(0)) = A(0, A_0^{-\alpha} A_0^\alpha u_0) = A(0, u_0) = A_0.$$

Theorem 3.1. *If the assumptions (A1)–(A7) are satisfied then there exists at least one continuously differentiable solution $u(t)$ of Eq. (1.1).*

Proof. To study the existence problem, we must introduce a set $\mathcal{B}$ of function $u(t)$, for $t \in J$ and a transformation $z_u = \Omega u$ defined as $z_u = A_0^\alpha z$, where z is the unique solution (see Pazy [11], 6.3.1) of the equation

$$\left.\begin{aligned}
&\frac{\partial}{\partial t}[z + g_u(t)] + A_u(t) z = f(t, A_0^{-\alpha} u(t)), \\
&z(0) \qquad\qquad\qquad\quad = u_0, \\
&z(\xi_k) \qquad\qquad\qquad\;\; = b_k(\tau_k) A_0^{-\alpha} u(\xi_k^-), \; k = 1, 2, \ldots
\end{aligned}\right\} . \tag{3.1}$$

We show that Ω has a fixed point, that is, there is a function $y \in \mathcal{B}$ such that $\Omega y = y$ and so $u = A_0^{-\alpha} y$ is the required solution of our Eq. (1.1).

Define the set

$$\mathcal{B} = \{u \in \mathbb{E} : \|u(t) - u(\tau)\| \leq C_{11}|t-\tau|^{\eta}, \text{ for } t, \tau \in J, \ u(0) = A_0^{\alpha}u_0\},$$

where η is any number satisfying $0 < \eta < \beta - \alpha$ and $\mathbb{E}$ is a Banach space $\mathcal{PC}(J : \mathbb{X})$ with usual sup norm. From assumption (A6), and the definition of $\mathcal{B}$ it follows that if T is sufficiently small (depending on $C_9, \eta, \|A_0^{\alpha}u_0\|$), then $\|u(t)\|_{\mathcal{C}} < r$, for $t \in J$.

Since $A_u(t)$ is well defined, it satisfies the condition

$$\begin{aligned}
\| \ (A_u(t) - A_u(\tau))A_0^{-1} &= C_2[|t-\tau|^{\epsilon} + \|u(t)-u(\tau)\|^{\rho}_{\mathcal{PC}}] \ \| \\
&\leq C_2[|t-\tau|^{\epsilon} + C_{11}|t-\tau|^{\eta\rho}] \\
&\leq C_{12}|t-\tau|^{\mu},
\end{aligned}$$

where $\mu = \min\{\epsilon, \eta\rho\}$ and it follows that for every $t \in J$ and λ with $Re\lambda \leq 0$,

$$\begin{aligned}
\|[\lambda I - A_u(t)]^{-1}\| &\leq C_{13}[|\lambda| + 1]^{-1} \\
\|[A_u(t) - A_u(\tau)]A_u^{-1}(s)\| &\leq C_{14}|t-\tau|^{\mu}, \text{ for every } \ t, \tau, s \in J.
\end{aligned}$$

By the assumptions, there exists a fundamental solution $S_u(t,s)$ corresponding to $A_u(t)$, and all estimates for the fundamental solutions derived in Theorem 2.1 hold uniformly with respect to $u \in \mathcal{B}$. Since, $f_u(0) = f(0, A_0^{-\alpha}u(0))$ and $g_u(0) = g(0, A_0^{-\alpha}u(0))$ is independent of u, we have from the above inequalities $\|f_u(t)\| \leq M_3$ and $\|g_u(t)\| \leq M_4$, where $M_3 > 0$ and $M_4 > 0$ from lemma (3.1) and using (K3) & (K4) we get

$$\begin{aligned}
&\|\Big[\sum_{k=0}^{\infty}\sum_{i=0}^{k}\prod_{j=i}^{k} b_j(\tau_j)A_0^{\alpha}\Big(\int_{\xi_{i-1}}^{\xi_i} S_u(t_1,s)f_u(s)ds - \int_{\xi_{i-1}}^{\xi_i} S_u(t_2,s)f_u(s)ds\Big)\Big]\| \\
&\quad + \|\Big[\sum_{k=0}^{\infty} A_0^{\alpha}\Big(\int_{\xi_k}^{t_1} S_u(t_1,s)f_u(s)ds - \int_{\xi_k}^{t_2} S_u(t_2,s)f_u(s)ds\Big)\Big]\| \\
&\quad - \|\Big[\sum_{k=0}^{\infty}\sum_{i=0}^{k}\prod_{j=i}^{k} b_j(\tau_j)A_0^{\alpha}\Big(\int_{\xi_{i-1}}^{\xi_i} A(s,u)S_u(t_1,s)g_u(s)ds \\
&\quad - \int_{\xi_{i-1}}^{\xi_i} A(s,u)S_u(t_2,s)g_u(s)ds\Big)\Big]\| \\
&\quad - \|\Big[\sum_{k=0}^{\infty} A_0^{\alpha}\Big(\int_{\xi_k}^{t_1} A(s,u)S_u(t_1,s)g_u(s)ds - \int_{\xi_k}^{t_2} A(s,u)S_u(t_2,s)g_u(s)ds\Big)\Big]\| \\
&\quad - \|\Big[\sum_{k=0}^{\infty}\prod_{j=1}^{k} b_j(\tau_j)A_0^{\alpha}\Big(g_u(t_1) - g_u(t_2)\Big)\Big]\| \\
&\leq \ M_3C_{15}|t_1-t_2|^{1-\alpha}(t_2-\xi_k)^{-1} + C_{16}|t_1-t_2|^{1-\alpha}[|log(t_1-t_2)|+1] \\
&\quad - \ M_4C_{17}|t_1-t_2|^{1-\alpha}(t_2-\xi_k)^{-1} - C_{18}|t_1-t_2|^{1-\alpha}[|log(t_1-t_2)|+1] \\
&\quad - \ M_1C_3|t_1-t_2|^{\epsilon}.
\end{aligned}$$

We shall check that the operator $\Omega : \mathcal{B} \to \mathbb{E}$ defined by

$$\left.\begin{aligned}\Omega u(t) = &\sum_{k=0}^{\infty}\prod_{j=1}^{k} b_j(\tau_j)A_0^{\alpha}S_u(t,0)[u_0 - g(0,u(0))]\\ &- \sum_{k=0}^{\infty}\prod_{j=1}^{k} b_j(\tau_j)A_0^{\alpha}.g(t,u(t))\\ &- \sum_{k=0}^{\infty}\sum_{i=1}^{k}\prod_{j=i}^{k} b_j(\tau_j)A_0^{\alpha}\int_{\xi_{i-1}}^{\xi_i} S_u(t,s)A(s,u)g(s,u(s))ds\\ &- \sum_{k=0}^{\infty} A_0^{\alpha}\int_{\xi_k}^{t} S_u(t,s)A(s,u)g(s,u(s))ds\\ &+ \sum_{k=0}^{\infty}\sum_{i=1}^{k}\prod_{j=i}^{k} b_j(\tau_j)A_0^{\alpha}\int_{\xi_{i-1}}^{\xi_i} S_u(t,s)f(s,u(s))ds\\ &+ \sum_{k=0}^{\infty} A_0^{\alpha}\int_{\xi_k}^{t} S_u(t,s)f(s,u(s))ds,\end{aligned}\right\} \tag{3.2}$$

has a fixed point. This fixed point is the solution of Eq. (1.1). Here $\mathcal{B}$ is closed convex and bounded set of $\mathbb{E}$. First we prove that Ω maps $\mathcal{B}$ into itself. Obviously $\Omega u(0) = A_0^{\alpha}u_0$. For any $0 \le t_1 \le t_2 \le T$, $0 \le \alpha \le \beta \le 1$, we get

$$\begin{aligned}&\|\Omega u(t_1) - \Omega u(t_2)\|\\ &= \sum_{k=0}^{\infty}\prod_{j=1}^{k}\|b_j(\tau_j)\|\|A_0^{-\beta}(u_0 - g(0,u(0)))\|\|A_0^{\alpha}[S_u(t_1,0) - S_u(t_2,0)]A_0^{\beta}\|\\ &\quad + \sum_{k=0}^{\infty}\prod_{j=1}^{k}\|b_j(\tau_j)\|\|A_0^{\alpha}.[g(t_1,u(t_1)) - g(t_2,u(t_2))]\| + \sum_{k=0}^{\infty}\sum_{i=1}^{k}\|\prod_{j=i}^{k} b_j(\tau_j)\|\\ &\quad \times \|A_0^{\alpha}\int_{\xi_{i-1}}^{\xi_i}[S_u(t_1,s)A(s,u)g(s,u(s)) - S_u(t_2,s)A(s,u)g(s,u(s))]ds\|\\ &\quad + \sum_{k=0}^{\infty}\|A_0^{\alpha}[\int_{\xi_k}^{t_1} S_u(t_1,s)A(s,u)g(s,u(s))ds - \int_{\xi_k}^{t_2} S_u(t_2,s)A(s,u)g(s,u(s))ds]\|\\ &\quad + \sum_{k=0}^{\infty}\sum_{i=1}^{k}\|\prod_{j=i}^{k} b_j(\tau_j)\|\|A_0^{\alpha}\int_{\xi_{i-1}}^{\xi_i}[S_u(t_1,s) - S_u(t_2,s)]f(s,u(s))ds\|\\ &\quad + \sum_{k=0}^{\infty}\|A_0^{\alpha}[\int_{\xi_k}^{t_1} S_u(t_1,s)f(s,u(s))ds - \int_{\xi_k}^{t_2} S_u(t_2,s)f(s,u(s))ds]\|.\end{aligned}$$

For sufficiently small T,

$$\begin{aligned}\|\Omega u(t_1) - \Omega u(t_2)\| \le\ & rC_{19}|t_1 - t_2|^{\beta-\alpha} + M_2C_{20}|t_1 - t_2|^{1-\alpha}(t_2 - \xi_k)^{-1}\\ &+ C_{21}|t_1 - t_2|^{1-\alpha} + M_3C_{22}|t_1 - t_2|^{1-\alpha}(t_2 - \xi_k)^{-1}\\ &+ C_{23}|t_1 - t_2|^{1-\alpha} + M_1C_3|t_1 - t_2|^{\epsilon},\end{aligned}$$

for $\eta < \beta - \alpha$ and $k = 1, 2, \ldots$. Hence Ω maps $\mathcal{B}$ into itself.

Next we prove this operator is continuous on the space $\mathbb{E}$. Let $u_1, u_2 \in \mathcal{B}$ and set $z_1 = A_0^{-\alpha}\Omega u_1, z_2 = A_0^{-\alpha}\Omega u_2$, then

$$\left.\begin{aligned} \frac{\partial}{\partial t}[z_m + g_{u_m}(t)] + A_{u_m}(t)z_m &= f_{u_m}(t), \\ z_m(0) &= u_0, \quad m = 1, 2, \dots \\ z(\xi_k) &= b_k(\tau_k)A_0^{-\alpha}u_m(\xi_k^-), \ k = 1, 2, \dots \end{aligned}\right\}. \tag{3.3}$$

Therefore,

$$\begin{aligned} &\frac{\partial}{\partial t}[z_1 - z_2 + g_{u_1}(t) - g_{u_2}(t)] + A_{u_1}(t)(z_1 - z_2) \\ &\quad = [A_{u_2}(t) - A_{u_1}(t)]z_2 + f_{u_1}(t) - f_{u_2}(t). \end{aligned}$$

It is easy to see that the function $A_{u_1}(t)z_2(t)$ and $A_0A_{u_2}^{-1}$ are uniformly Holder continuous, and so $A_0z_2(t) = [A_0A_{u_2}^{-1}]A_{u_1}(t)z_2(t)$ is uniformly Holder continuous. Similarly the functions $f_{u_1}(t) - f_{u_2}(t)$ and $g_{u_1}(t) - g_{u_2}(t)$, are also uniformly Holder continuous in $[\delta, T], \delta > 0$. Hence we have

$$\begin{aligned} &z_1(t) - z_2(t) \\ &\quad = \sum_{k=0}^{\infty}\prod_{j=1}^{k} b_j(\tau_j)S_{u_1}(t,\delta)[z_1(\delta) - z_2(\delta)] + [g_{u_1}(0) - g_{u_2}(0)] \\ &\qquad - \sum_{k=0}^{\infty}\prod_{j=1}^{k} b_j(\tau_j)[g_{u_1}(\delta) - g_{u_2}(\delta)] \\ &\qquad - \sum_{k=0}^{\infty}\sum_{i=1}^{k}\prod_{j=i}^{k} b_j(\tau_j)\int_{\xi_{i-1}}^{\xi_i} A_{u_1}(s)S_{u_1}(t,s)[g_{u_1}(s) - g_{u_2}(s)]ds \\ &\qquad - \sum_{k=0}^{\infty}\int_{\xi_k}^{t} A_{u_1}(s)S_{u_1}(t,s)[g_{u_1}(s) - g_{u_2}(s)]ds \\ &\qquad + \sum_{k=0}^{\infty}\sum_{i=1}^{k}\prod_{j=i}^{k} b_j(\tau_j)\int_{\xi_{i-1}}^{\xi_i} S_{u_1}(t,s) \\ &\qquad \times \Big([A_{u_2}(s) - A_{u_1}(s)]z_2(s) + [f_{u_1}(s) - f_{u_2}(s)]\Big)ds \\ &\qquad + \sum_{k=0}^{\infty}\int_{\xi_k}^{t} S_{u_1}(t,s)[A_{u_2}(s) - A_{u_1}(s)]z_2(s) + [f_{u_1}(s) - f_{u_2}(s)]ds. \end{aligned}$$

Since $A_0^{\alpha}\int_0^t S_{u_2}(t,s)f_{u_2}(s)ds$ is bounded, it follows that $\|A_0z_2(t)\| \le C_{24}T^{\beta-\alpha}$.

Hence we can take $\delta \to 0$ in the above equation, we have

$$\begin{aligned} z_1(t) - z_2(t) = &\sum_{k=0}^{\infty}\prod_{j=1}^{k} b_j(\tau_j)S_{u_1}(t,0)[g_{u_1}(0) - g_{u_2}(0)] \\ &- \sum_{k=0}^{\infty}\sum_{i=1}^{k}\prod_{j=i}^{k} b_j(\tau_j)\int_{\xi_{i-1}}^{\xi_i} A_{u_1}(s)S_{u_1}(t,s)[g_{u_1}(s) - g_{u_2}(s)]ds \end{aligned}$$

$$
\begin{aligned}
&- \sum_{k=0}^{\infty} \int_{\xi_k}^{t} A_{u_1}(s) S_{u_1}(t,s)[g_{u_1}(s) - g_{u_2}(s)]ds \\
&+ \sum_{k=0}^{\infty} \sum_{i=1}^{k} \prod_{j=i}^{k} b_j(\tau_j) \\
&\times \int_{\xi_{i-1}}^{\xi_i} S_{u_1}(t,s)\Big([A_{u_2}(s) - A_{u_1}(s)]z_2(s) + [f_{u_1}(s) - f_{u_2}(s)]\Big)ds \\
&+ \sum_{k=0}^{\infty} \int_{\xi_k}^{t} S_{u_1}(t,s)[A_{u_2}(s) - A_{u_1}(s)]z_2(s) + [f_{u_1}(s) - f_{u_2}(s)]ds.
\end{aligned}
$$

Since $z_1 = A_0^{-\alpha} \Omega u_1$ and $z_2 = A_0^{-\alpha} \Omega u_2$ and from (A3)–(A6), it follows that

$$
\begin{aligned}
&\|\Omega u_1(t) - \Omega u_2(t)\| \\
&\leq \sum_{k=0}^{\infty} \prod_{j=1}^{k} \|b_j(\tau_j)\| \|A_0^{\alpha} S_{u_1}(t,0)[g_{u_1}(0) - g_{u_2}(0)]\| \\
&\quad + \sum_{k=0}^{\infty} \sum_{i=1}^{k} \prod_{j=i}^{k} \|b_j(\tau_j)\| \int_{\xi_{i-1}}^{\xi_i} \|A_0^{\alpha} A_{u_1}(s) S_{u_1}(t,s)[g_{u_1}(s) - g_{u_2}(s)]\| ds \\
&\quad + \sum_{k=0}^{\infty} \int_{\xi_k}^{t} \|A_0^{\alpha} A_{u_1}(s) S_{u_1}(t,s)[g_{u_1}(s) - g_{u_2}(s)]\| ds \\
&\quad + \sum_{k=0}^{\infty} \sum_{i=1}^{k} \prod_{j=i}^{k} \|b_j(\tau_j)\| \int_{\xi_{i-1}}^{\xi_i} \|A_0^{\alpha} S_{u_1}(t,s)[A_{u_2}(s) - A_{u_1}(s)]z_2(s)\| ds \\
&\quad + \sum_{k=0}^{\infty} \sum_{i=1}^{k} \prod_{j=i}^{k} \|b_j(\tau_j)\| \int_{\xi_{i-1}}^{\xi_i} \|A_0^{\alpha} S_{u_1}(t,s)[f_{u_1}(s) - f_{u_2}(s)]\| ds \\
&\quad + \sum_{k=0}^{\infty} \int_{\xi_k}^{t} \|A_0^{\alpha} S_{u_1}(t,s)[A_{u_2}(s) - A_{u_1}(s)]z_2(s) + [f_{u_1}(s) - f_{u_2}(s)]\| ds \\
&\leq M_1 \|A_0^{\alpha}[S_{u_1}(t,0) - S_{u_1}(\tau,0)]\| \|g_{u_1}(0) - g_{u_2}(0)\| \\
&\quad + M_1 \|A_0^{\alpha} S_{u_1}(\tau,0)\| \|g_{u_1}(0) - g_{u_2}(0)\| \\
&\quad + C_{10}|t - \xi_k|^{1-\alpha}(|log(t - \xi_k)| + 1)\|u_1 - u_2\|^{\rho} \\
&\quad + C_{11}|t - \xi_k|^{1-\alpha}(|log(t - \xi_k)| + 1)\|u_1 - u_2\|^{\rho} + M_1 C_2 \|u_1 - u_2\|^{\rho} |t - \xi_k|^{\mu} \\
&\leq \Big(C_{25}|t - \tau|^{\beta-\alpha} + C_{26}|t - \xi_k|^{1-\alpha}(|log(t - \xi_k)| + 1) + C_{27}|t - \xi_k|^{\mu}\Big) \\
&\quad \times \|u_1 - u_2\|^{\rho}.
\end{aligned} \tag{3.4}
$$

This shows that $\Omega : \mathcal{B} \to \mathbb{E}$ is continuous. We shall prove that this operator is completely continuous. We now claim that the set $\Omega\mathcal{B}$ is contained in a compact subset of $\mathbb{E}$. Also, the function $u(t)$ of $\mathcal{B}$ is uniformly bounded and equicontinuous. By Arzela–Ascoli's theorem, it is sufficient to show that, for each t the set $\{\Omega u(t) : u \in \mathcal{B}\}$ is contained in a compact subset of $\mathbb{X}$. For each $t \in [0, T]$, we can write $\Omega u(t) = A_0^{-\omega} A_0^{\omega} \Omega u(t)$, $(0 < \omega < \beta - \alpha)$. Since $\{A_0^{\omega} \Omega u(t) : u \in \mathcal{B}\}$ is a bounded subset of $\mathbb{X}$, and since $A_0^{-\omega}$ is completely continuous, it follows that the set $\{\Omega u(t) : u \in \mathcal{B}\}$ is contained in a compact subset of $\mathbb{X}$. Therefore, by the

Schauder fixed point theorem, Ω has a fixed point $z \in \mathcal{B}$ such that $\Omega z(t) = z(t)$ which is a solution of Eq. (3.1) satisfies

$$\begin{aligned}
z(t) = {} & \sum_{k=0}^{\infty} \prod_{j=1}^{k} b_j(\tau_j) A_0^{\alpha} S_z(t,0)[u_0 - g_z(0)] - \sum_{k=0}^{\infty} \prod_{j=1}^{k} b_j(\tau_j) A_0^{\alpha} g_z(t) \\
& - \sum_{k=0}^{\infty} \sum_{i=1}^{k} \prod_{j=i}^{k} b_j(\tau_j) A_0^{\alpha} \int_{\xi_{i-1}}^{\xi_i} S_z(t,s) A(s,u,) g_z(s) ds \\
& + \sum_{k=0}^{\infty} A_0^{\alpha} \int_{\xi_k}^{t} S_z(t,s) f_z(s) ds \\
& - \sum_{k=0}^{\infty} A_0^{\alpha} \int_{\xi_k}^{t} S_z(t,s) A(s,u) g_z(s) ds \\
& + \sum_{k=0}^{\infty} \sum_{i=1}^{k} \prod_{j=i}^{k} b_j(\tau_j) A_0^{\alpha} \int_{\xi_{i-1}}^{\xi_i} S_z(t,s) f_z(s) ds.
\end{aligned}$$

Then $u(t) = A_0^{-\alpha} z(t)$ satisfies

$$\begin{aligned}
u(t) = {} & \sum_{k=0}^{\infty} \prod_{j=1}^{k} b_j(\tau_j) S_{A_0^{\alpha} u}(t,0)[u_0 - g_{A_0^{\alpha} u}(0)] - \sum_{k=0}^{\infty} \prod_{j=1}^{k} b_j(\tau_j) g_{A_0^{\alpha} u}(t) \\
& - \sum_{k=0}^{\infty} \sum_{i=1}^{k} \prod_{j=i}^{k} b_j(\tau_j) \int_{\xi_{i-1}}^{\xi_i} S_{A_0^{\alpha} u}(t,s) A(s,u) g_{A_0^{\alpha} u}(s) ds \\
& - \sum_{k=0}^{\infty} \int_{\xi_k}^{t} S_{A_0^{\alpha} u}(t,s) A(s,u) g_{A_0^{\alpha} u}(s) ds \\
& + \sum_{k=0}^{\infty} \sum_{i=1}^{k} \prod_{j=i}^{k} b_j(\tau_j) \int_{\xi_{i-1}}^{\xi_i} S_{A_0^{\alpha} u}(t,s) f_{A_0^{\alpha} u}(s) ds + \sum_{k=0}^{\infty} \int_{\xi_k}^{t} S_{A_0^{\alpha} u}(t,s) f_{A_0^{\alpha} u}(s) ds.
\end{aligned}$$

Thus, $u(t)$ is the solution of our Eq. (1.1). Hence the proof is completed. □

Theorem 3.2 ([5]). *If the assumptions (A1)–(A7) are satisfied with $\rho = 1$ then the assertion of Theorem 3.1 is valid and the solution is unique.*

Proof. If $\rho = 1$, then from Eq. (3.4), shows that for sufficiently small T, Ω is Lipschitz continuous which is a particular case of Holder continuous, that is, $\|\Omega u_1(t) - \Omega u_2(t)\| \le \mathcal{N} \|u_1 - u_2\|$ for some $\mathcal{N} < 1$. Then by the Banach contraction principle, Ω has a unique fixed point. □

4. Application

The purpose of this section is to provide an example to show applications of our obtained results.

Example 4.1. Consider the following nonlinear parabolic random impulsive differential equation

$$\left.\begin{aligned} &\frac{\partial u}{\partial t}+\sum_{|\alpha|=2n} a_\alpha(x,t,u,Du,\ldots,D^{2n-1}u)D^\alpha u = f(x,t,u,Du,\ldots,D^{2n-1}u),\\ &\qquad \frac{\partial^j u}{\partial v^j}=0 \text{ on } S_T=\{(x,t): x\in\partial\Omega,\ 0\le t\le T\},\ 0\le j\le n-1,\\ &\quad u(x,0)=0 \text{ on } \Omega_0=\{(x,0): x\in\partial\Omega\},\\ &\quad u(\xi_k)=q_k(\tau_k)u(\xi_k^-),\ k=1,2,\ldots, \end{aligned}\right\} \tag{4.1}$$

in a cylinder $Q_T = \Omega \times (0,T)$ with coefficients in $\bar{Q}_T$, where Ω is a bounded domain in $\mathbb{R}^n$, $\partial\Omega$ the boundary of Ω, v is the outward normal. Here the parabolicity means that for any vector $z \neq 0$ and for arbitrary values of $x, t, u, Du, \ldots, D^{2n-1}u$,

$$(-1)^n \mathrm{Re}\{\sum_{|\alpha|=2n} a_\alpha(x,t,u,Du,\ldots,D^{2n-1}u)z^\alpha\} \ge C|z|^{2m},\ C>0.$$

Assume that τ_k is the random variable defined on $D_k \equiv (0,d_k)$ for $k = 1,2,\ldots$, where $0 < d_k < +\infty$. Further, assume that τ_i and τ_j are independent of each other as $i \neq j$ for $i,j = 1,2,\ldots$; $\xi_0 = t_0$; $\xi_k = \xi_{k-1} + \tau_k$, for $k = 1,2,\ldots$ and $\max_{i,k}\prod_{j=i}^{k}\|q_k(\tau_j)\|^2 < \infty$. Here $t_0 \in \mathbb{R}_\eta$ is an arbitrarily given real number.

If $u_0(x) \in C^{2n-1}(\bar{\Omega})$, then $A_0 u = \sum_{|\alpha|=2n} a_\alpha(x,t,u,Du,\ldots,D^{2n-1}u)D^\alpha u$ is a strongly elliptic operator with continuous coefficients. So the condition (A1) holds. Let us take $\mathbb{X}$ to be $L^p(\Omega)$, $1 < p < \infty$. Then A_0^{-1} maps bounded subsets of $L^p(\Omega)$ in to bounded subsets of $W^{2n,p}(\Omega)$, so it is a completely continuous operator in $L^p(\Omega)$. Further, if $(2n-1)/2n < \alpha < 1$, then $|D^\beta A_0 - \alpha u|_{0,p}^{\Omega} \le C|u|_{0,p}^{\Omega}$, $\ 0 \le |\beta| \le 2n-1$, where C depends only on a bound on the coefficients A_0, on a module of strong ellipticity and on a modulus of continuity of the leading coefficients. Here the norm is defined as

$$|u|_{0,p}^{\Omega} = \{\sum_{|\alpha|\le j}\int_\Omega |D^\alpha u(x)|^p dx\}^{\frac{1}{p}}$$

for any nonnegative integer j and a real number p, $1 \le p < \infty$. It follows that if f and a_α are continuously differentiable in all variables, then (A3)–(A4) hold with $\sigma = \rho = 1$. Hence there exist fundamental operator solution $S_x(t,s)$ for Eq. (4.1). The nonlinear function f satisfies the conditions (A4)–(A6). Hence by the above theorem there exists a local solution for Eq. (4.1).

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